State Parameter and Undrained Shear Strength of Mine Tailings from In-situ and Laboratory Testing

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ABSTRACT: Determination of the in-situ state parameter and the undrained shear strength of tailings is challenging in practice because of the inherent uniqueness of these materials as it relates to their variability and lack of structure due to their depositional conditions. As a result, these aspects complicate the formulation and characterization of the tailings within the framework of critical state of soil mechanics (CSSM) compared to natural soils. The authors have developed correlations that use different in-situ testing techniques and index properties, which facilitate the characterization of mine tailings. Further, laboratory testing was performed to determine characteristic CSSM parameters, notably λ_{10} and Γ. The combination of laboratory and in-situ testing is then used to determine the strength and state parameters of mine tailings. This paper presents the correlations between in-situ testing and index properties that facilitate the characterization of mine tailings and the procedures that use these correlations and laboratory testing results to estimate the in-situ state parameter and the undrained shear strength of the mine tailings.

1 INTRODUCTION

Mine tailings are the waste material from mineral processing (i.e., crushing, grinding, and separation) of ore and are conventionally slurry discharged in the tailings storage facility (TSF). Mine tailings consist of material with particle sizes ranging from coarse (i.e., sand-size) to fine (i.e., silt-size to clay-size). Slurry tailings tend to exhibit contractive behavior and thus are susceptible to liquefaction due to their method of deposition, geologic age, and stress history of the deposit (Terzaghi et al., 1996). These depositional conditions and material characteristics generate high variability of tailings deposits and properties, which pose technical challenges for determining state parameters and the undrained shear strength (Contreras and Harvey, 2021).

Fine tailings are recent deposits of a very young geologic age and therefore have not experienced significant aging or weathering to develop a robust soil structure. And, because the fine tailings are deposited hydraulically, the material comes into equilibrium in a very loose condition with a high void ratio. Generally, the fine tailings do not experience loading other than from continued tailings deposition and dam construction, resulting in nearly normally consolidated deposits. Furthermore, the fine tailings commonly contain non-plastic or low plasticity solids. These factors create very difficult conditions for collecting truly “undisturbed” samples for laboratory shear strength testing. This is particularly significant because the undrained shear strength is largely controlled by the incipient soil fabric and void ratio, which can be easily disturbed by conventional sampling methods, transportation, handling, and preparation (Viana Da Fonseca et al., 2015).

However, laboratory shear strength testing still serves a valuable purpose for characterizing and understanding the overall behavior of the fine tailings. Laboratory testing allows for a more controlled environment and boundary conditions during testing. In mine tailings, laboratory testing is limited to specimens reconstituted from slurry and consolidated to water contents representative of those in the field. Specimens in the laboratory can be prepared to void ratios similar to those in the field even though the soil fabric may or may not be similar as it is much more difficult to duplicate the in-situ soil fabric. The use of reconstituted specimens from slurry or moist-tamping methods for assessing tailings within the framework of CSSM is of less concern since the objective is to determine the fine tailings behavior at the critical state (i.e., very large strains) when any in-situ soil fabric has been destroyed.

In contrast with laboratory testing methods, in-situ testing offers several advantages and some limitations that must be properly considered. Beginning with the advantages, CPT and FVT do not require the collection of representative undisturbed samples for subsequent laboratory testing; thus, issues related to sample disturbance are eliminated. Additionally, the deposit is more comprehensively and efficiently characterized compared to laboratory testing of discrete samples because CPT soundings produce near-continuous data profiles with depth. FVT soundings typically include multiple tests performed successively with depth. As a result, the in-situ testing provides a better representation of the material variability and may identify particularly strong or weak layers that may go undetected by traditional sampling methods. Finally, CPT and FVT are generally more repeatable and reliable than other methods due to the standardization of equipment and procedures.

However, in-situ testing also presents some limitations for controlling testing conditions, which are typically afforded a high level of control within the laboratory environment. For example, the state of stress and drainage conditions cannot be controlled or modified during in-situ testing and, in many cases, may not even be well known. This is particularly critical because the strain rate used (i.e., cone penetration rate and vane rotation rate) directly affects whether the material behaves in a drained, partially drained, or undrained manner during the test. Recognizing these limitations, the authors have performed analyses and evaluations to better understand these aspects and/or to modify in-situ testing procedures to account for them, specifically with regard to fine tailings (Contreras and Harvey, 2021).

Recognizing the advantages and limitations of both in-situ and laboratory testing of fine tailings, the authors have found that both techniques should be combined to comprehensively assess the characteristics and behavior of the fine tailings within the framework of the CSSM. As such, the authors have developed an approach to estimate the state parameter and undrained shear strength of fine tailings using both in-situ and laboratory testing techniques. This paper contains a detailed description of the proposed approach and an example of its application. It briefly describes the CSSM concepts, challenges in fine tailings, the proposed approach for state parameter and undrained shear strength ratio (USSR) determination, an example of application, and a comparison.

2 CSSM FRAMEWORK

In simple terms, CSSM is an effective stress framework that describes mechanical soil response by tying compressibility and shear behavior. It originated in the work of Casagrande (1936) when he introduced the concept of "critical void ratio." He noticed that under the same consolidation stress, in direct shear tests, the ultimate strength of the sand and its void ratio (i.e., large displacement) were the same regardless of its initial density. The shear displacement of the sand will depend on its initial density or void ratio with respect to the critical void ratio. Loose sand with an initial void ratio higher than the critical void ratio will exhibit a contractive response. In contrast, dense sand with an initial void ratio lower than the critical void ratio will exhibit a dilative response.

This concept was subsequently verified experimentally and expanded (Hvorslev, 1960; Henkel, 1960; Parry, 1960; and Castro, 1969). Roscoe et al. (1958) and Schofield and Worth (1967) developed the mathematical formulation and final formalization of CSSM. Jefferies and Been (1985) introduced the term state parameter. This led to the formulation of constitutive models for implementation in different software packages such as CAM-CLAY, Modified CAM-CLAY, and NorSand, among others.

Figure 1 illustrates the framework of CSSM in terms of the consolidation and shear response of a normally consolidated soil in a triaxial compression test. There are similar formulations for other cases like over-consolidated soils, but this was thought to be more illustrative and representative for mine tailings. Figure 1 shows the stress-strain relationship for isotropic consolidated drained (CD) and undrained tests (CIU), stress paths of both tests in p′-q space, and the critical state line defined by λ_{10} and Γ parameters in e-log p′ space.

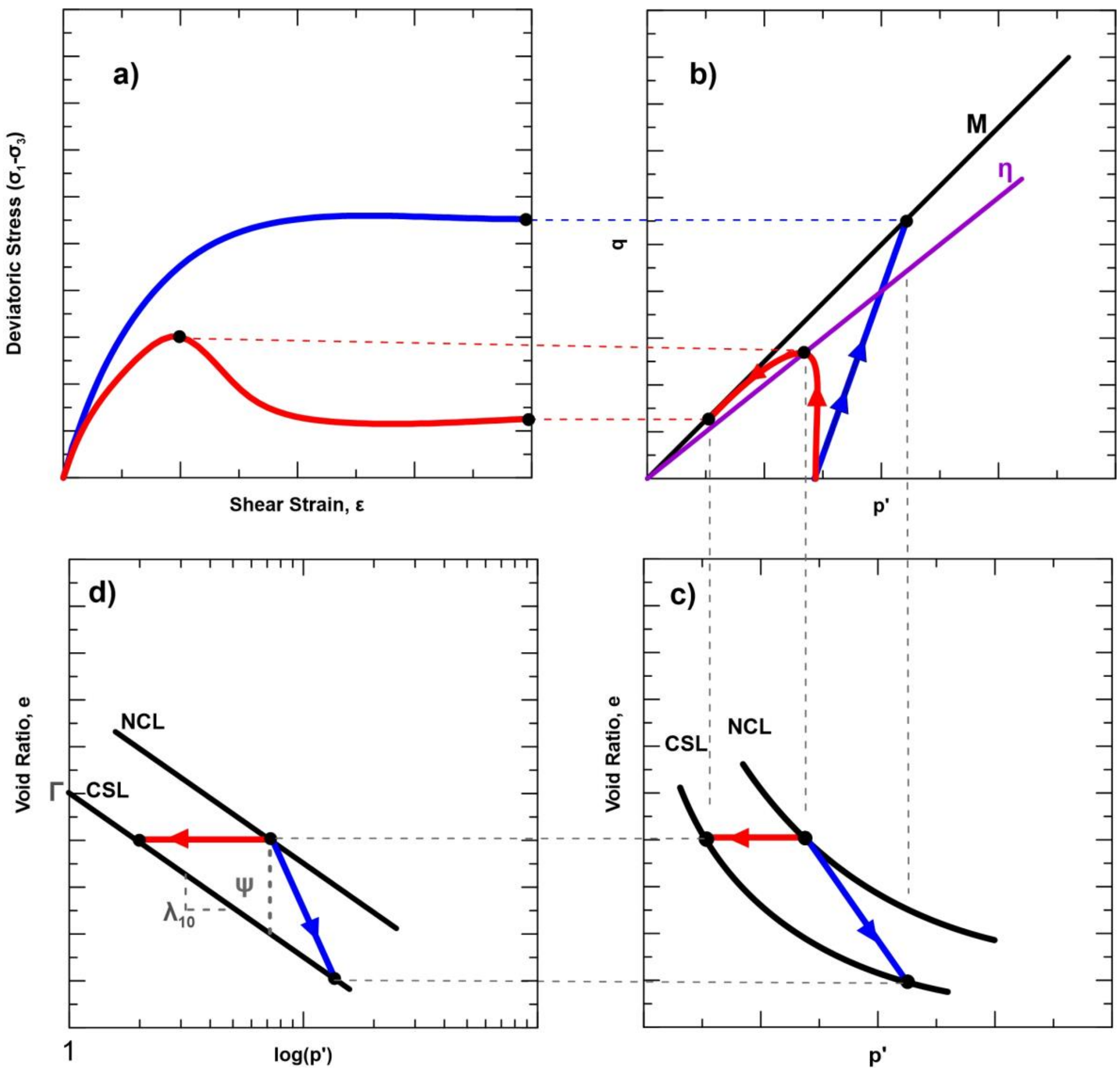

Figure 1. Shear (a, b) and consolidation (c, d) schematic of an NC soil in CSSM framework

Figure 1a shows the stress-strain relationships for the drained and undrained tests. The undrained test exhibits a peak (i.e., yield strength) at low strain and then drops to the steady-state or liquefied strength. Similarly, the drained test increases stress until it reaches its ultimate strength. At large strain, both the drained and undrained strengths fall into the critical state line (CSL) in the p′-q space (Fig. 1b) and the e-log p′ space (Fig. 1d).

Figure 1b shows the stress path in p′-q space for both tests and illustrates the contractive response of the material in the undrained test as a result of the positive shear-induced pore-water pressure. The drained test shows a continuous increase in q until it reaches the CSL, represented by the line of slope M. The undrained test stress path shows the instability line, η, which represents the yield undrained shear strength or the locus of points where instability occurs during shearing resulting in rapid strength loss until it reaches the CSL.

Figure 1c shows the CSL in an e-p′ space and the path of a soil in the normally consolidated line (NCL) to the CSL during drained and undrained tests. In the undrained test, the void ratio remains constant between the NCL and the CSL as there is no volume change. In the drained test, the void ratio decreases as volume changes with associated stress increase. Figure 1d shows the characteristics of CSSM parameters that define the CSL, notably λ_{10} and Γ. Finally, Figure 1d shows the state parameter, Ψ, which denotes the difference in void ratio at a given effective stress between the material void ratio, e, and the void ratio at the CSL.

3 CHALLENGES IN TAILINGS CHARACTERIZATION

Starting with the generation of the tailings, which requires crushing, grinding, and separation of ore (generally involving hard rock), mineral processing is a complex process that reduces the material size until the liberation of the mineral-ore is achieved, generating material from sand to clay size with different characteristics. The majority may or may not necessarily be clay minerals; thus, plasticity and clay-size fraction (CF, defined as the percentage by weight particles less than 0.002 mm) are important parameters. Furthermore, the inherent variability of the ore body causes changes in the tailings generated during the mine life, which introduces additional changes in the tailings stream. As a result, it is common to have a significant amount of grain-size and other index property variations in a TSF, resulting in heavy interlayering and erratic distribution.

CSL is unique to a particular material and its properties (Poulos et al., 1985). The slope (λ_{10}) of the CSL is mainly affected by the shape of the grains in a given soil, while the vertical position or intercept (Γ) of the CSL is affected by small differences in the grain-size distribution. Studies by Vaid and Chern (1985), Vaid et al. (1990), Konrad (1990), Robertson (2017), and Sadrekarimi (2014) indicate that the CSL may also be influenced by the mode of shear and effective confining stress. Therefore, the large amount of size variation and interlayering create challenges in the assignment of the slope (λ_{10}) and intercept (Γ) of the CSL for the different materials within the fine tailings deposit. An infinite number of CSLs are expected within the TSF deposit; however, an index property, such as CF, may be used to bracket the boundaries of the variation.

Another challenge is the estimation of the state parameter of the material in-situ. While the in-situ water content along with the specific gravity (G_s) can be used to estimate the void ratio (in saturated materials), a challenge arises from the fact that the CSL for each material is different. Thus, assigning the correct CSL parameters for each point is susceptible to error.

The authors approached these difficulties by trying to correlate an index soil property characteristic to a direct measurement in the field using in-situ testing. This subsequently is correlated to CSSM parameters obtained in the laboratory via testing. The overall approach to overcoming these difficulties is explained in detail below and illustrated with an example.

4 PROPOSED APPROACH

The proposed approach uses laboratory and in-situ testing to estimate the state parameter and the undrained shear strength of the fine tailings. In general, the approach involves the following:

- **Step1**: Collect representative bulk samples of fine tailings from the TSF deposit so that it brackets a reasonably wide range of properties (i.e., grain size, CF, and plasticity). Collect enough material, as the same batch will be used for triaxial and laboratory vane shear tests.
- **Step 2**: Use reconstituted specimens to perform a series of triaxial tests to determine the CSL and, notably, the λ_{10} and Γ parameters. The tests to determine these parameters should include drained and undrained tests. The tests should be run to very large strains (i.e., greater than 25 percent) to ensure the critical state has been reached. At least six to eight tests should be used to determine each CSL. The same material should be reused in each test.

- **Step 3**: Use reconstituted specimens to perform a series of laboratory confined vane shear tests under different confining pressures and at different initial moisture contents (i.e., generating different state parameters) to measure the yield and remolded shear strength. Make sure to use the same material as in the CSL determination. See Harvey and Contreras (2022) for additional details regarding this procedure.
- **Step 4**: Use the CPT to estimate the in-situ void ratio and the associated CSL parameters for a given material, notably the λ_{10} and Γ. This step and its basis are described in detail later in this paper. The authors have developed a comprehensive correlation to estimate CF from ΔQ (Saye et al., 2017), similar to I_b from Robertson (2016).
- **Step 5**: Using the estimates of in-situ void ratio(s) and CSL parameter(s) from the prior step, estimate the state parameter for the materials in a given sounding.
- **Step 6**: Using the estimated state parameter and the correlation between the state parameter and USSR developed in Step 3, estimate the undrained shear strength ratio.

Details of the procedure are presented below. The procedure description uses actual data from a TSF where the authors are involved. A series of CPT soundings and adjacent soil borings were performed to collect disturbed samples for index testing and to develop correlations. The procedure illustrates the correlation developed as part of this approach. One CPT sounding is used to show the procedure. A comparison with a separate independent procedure for USSR determination is also presented.

5 LABORATORY TESTING

Laboratory testing consisted of index property testing, triaxial compression testing for CSL parameter determination, and laboratory vane shear tests. Four bulk samples of fine tailings were collected from the TSF. These four samples were selected based on the range of grain sizes, plasticity, and CF found in the deposit.

5.1 *Index Testing*

Table 1 summarizes the index properties of the representative samples for CSL and laboratory vane shear testing.

Table 1 Index Properties of Fine Tailings Used in Laboratory Testing Program

	Atterberg Limits (Casagrande ASTM D4318)						
Sample	Liquid Limit (%)	Plastic Limit (%)	Plasticity Index (%)	Clay Fraction (%)	Specific Gravity	Percentage of Coarser Material (%)	USGS Soil Type
Sample 1	40	25	15	30.6	2.98	79.8	Lean Clay (CL)
Sample 2	31	23	8	21.6	2.94	49.4	Lean Clay (CL)
Sample 3	22	NP	NP	14.6	2.94	30.8	Lean Silt (ML)
Sample 4	21	19	2	12.9	2.93	26.9	Lean Silt (ML)

In addition to the index testing of the bulk samples in Table 1, disturbed samples from the soil borings adjacent to the CPT soundings were also tested for water content, grain size, CF, plasticity, and specific gravity. These results were used in the development of the correlation presented later.

5.2 *CSL Parameter Determination*

Reconstituted specimens were used to perform a series of triaxial tests to determine the CSL and, notably, the λ_{10} and Γ parameters. The material was prepared using the moist-tamp method to achieve fairly loose conditions, and CIU and CD triaxial tests were conducted. The triaxial tests were run to a large strain, typically 30 percent, to ensure that the critical state was reached. For each sample (Table 1), a minimum of six to eight tests were run to determine each CSL.

A modular-based platen system, similar to the one developed by Reid and Fourie (2019), was manufactured for handling the specimen after completion of the test. The entire specimen was then placed in a freezer until frozen. At that time, the specimen was removed, weighed, and dried for the most accurate determination of the void ratio at the end of the test. This is not possible using standard equipment.

The same representative bulk samples were used throughout the laboratory testing program to minimize the potential for material variability. At the end of the laboratory testing program, the index properties were measured again to verify that no changes had occurred.

Table 2 represents the results of triaxial compression tests in terms of the CSL parameters Γ and λ_{10}, as well as their index properties.

Table 2 Index Property and Critical State Parameters from Triaxial Compression Testing

	Atterberg Limits (Casagrande ASTM D4318)					CSL Parameters		
Sample	Liquid Limit (%)	Plastic Limit (%)	Plasticity Index (%)	Clay Fraction (%)	Percent of Coarser Material (%)	λ_{10}	Γ	Soil Type
FT/S-1	31	23	8	21.6	49.4	0.107	0.99	Lean Clay (CL)
FT/S-2	40	25	15	30.6	79.8	0.186	1.34	Lean Clay (CL)
FT/S-3	21	19	2	12.9	26.9	0.107	1.05	Lean Silt (ML)
FT/S-3’	22	NP	NP	14.6	30.8	0.103	0.87	Lean Silt (ML)

The CSL parameters Γ (intercept) and λ_{10} (slope) are a representation of the compressibility and critical state of the material with respect to the void ratio and effective mean stress (Cambridge formulation). Material with an in-situ void ratio greater than the CSL will behave in a contractive manner. In contrast, a material with an in-situ void ratio less than the CSL will behave in a dilative manner. Figure 2 shows the CSL results in the e-σ'_m space and the correlation between λ_{10} (slope) and CF.

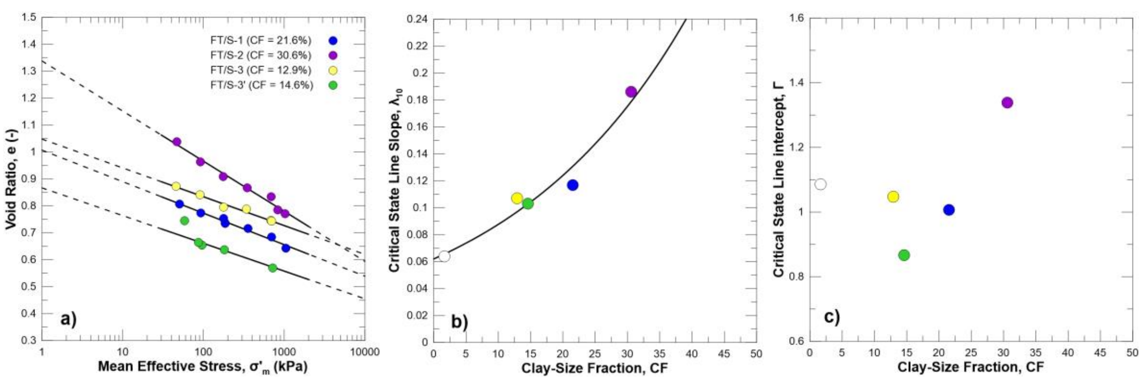

Figure 2. CSLs for tailings tested and correlation between slope, λ_{10}, intercept, Γ, and CF

5.3 *Laboratory Vane Shear Testing*

A series of laboratory vane shear tests were performed generally according to ASTM D4648 with modified equipment and procedures on reconstituted specimens of representative bulk samples of the fine tailings. The same material used in the CSL parameter determination testing was used in the LVT. A detailed description of the procedure associated with this testing is presented in Harvey and Contreras (2022). As a result, only final results and summary correlations are presented herein. Figure 3 shows the correlation between the yield and liquefied undrained shear strength ratio ($USSR_{YIELD}$ and $USSR_{LIQ}$) and the state parameter, Ψ.

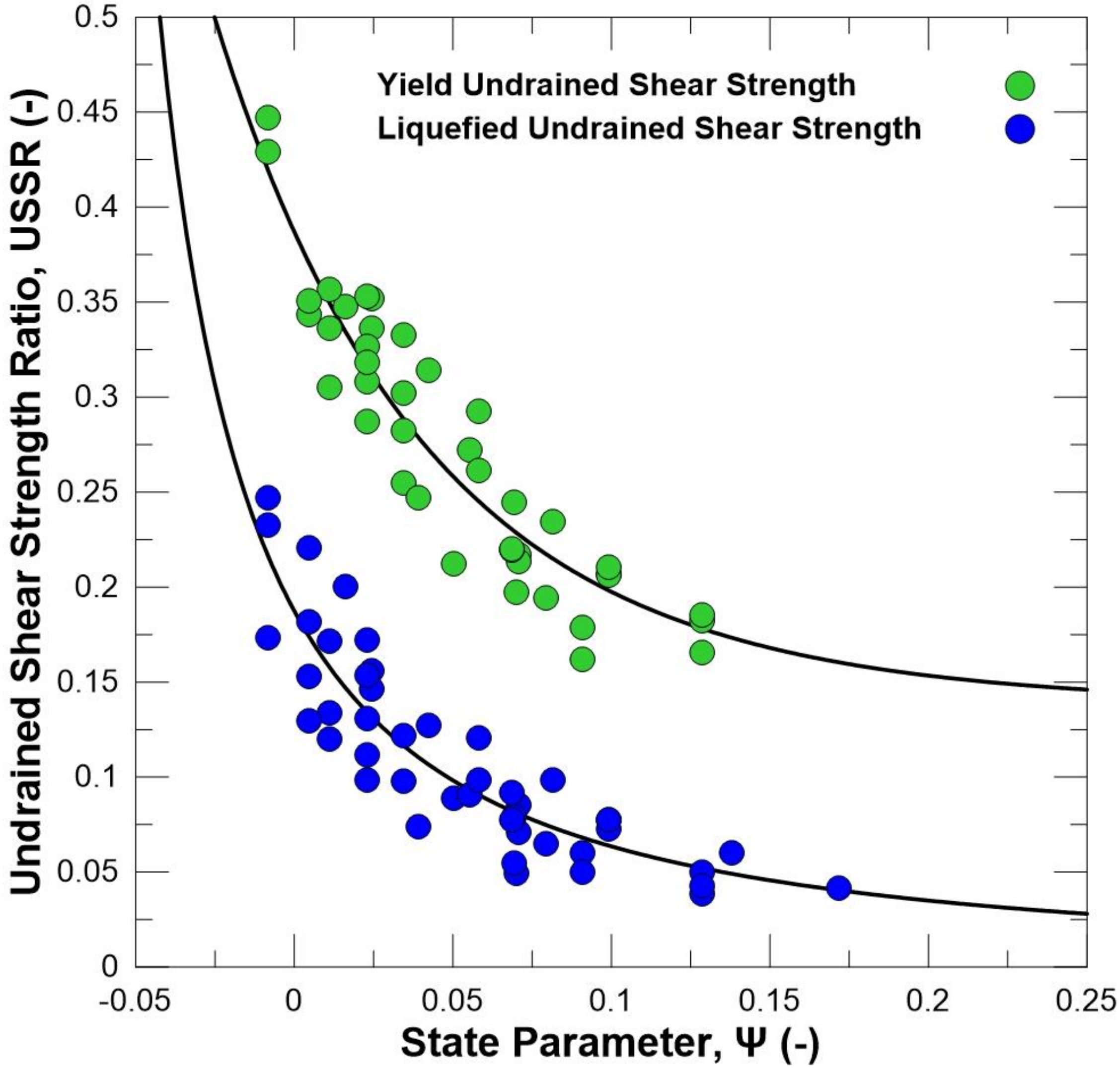

Figure 3. Correlation between USSR and state parameter

6 IN-SITU TESTING

As previously indicated, the in-situ testing involved a series of CPT soundings and soil borings adjacent to each other; nearly continuous disturbed samples were collected for index testing and correlation development. The in-situ and index testing for correlation development can be categorized into two groups. The first group is the CPT-index properties correlations in terms of CF and liquid limit. The second group is the CPT-void ratio correlation, which incorporates the impact of overburden pressure.

6.1 *CPT-Index Testing*

Saye et al. (2017) introduced the parameter ΔQ as an empirical parameter from the CPT for soil classification and improved correlation with soil index properties. ΔQ uses the overburden stress normalized net corrected tip stress, Q_t; the cone sleeve friction, f_s; and the effective vertical stress, σ'_{vo}. ΔQ is fairly similar to the modified behavior type index, I_b, proposed by Robertson (2016). In fact, both are associated with a more hyperbolic shape for soil behavior type boundaries in the *Log* Q_t–*Log* F_r space.

ΔQ was used as an index representing the CPT data, and it was then correlated to the CF of the tailings obtained from grain-size testing in the adjacent soil boring. Figure 4 shows the correlation between ΔQ and the CF developed by the authors. It shows that ΔQ varies from approximately 47 for low CF, decreases rapidly for a CF of up to 10 percent, and then gradually to approximately a ΔQ of 14 as the CF increases up to 43 percent. The correlation in Figure 4 applies only to fine tailings that are hydraulically deposited and thus essentially normally consolidated, fine tailings derived from hard rock ore, and fine tailings that are saturated. Data in Figure 4 contains information from three different TSFs from the same mining region.

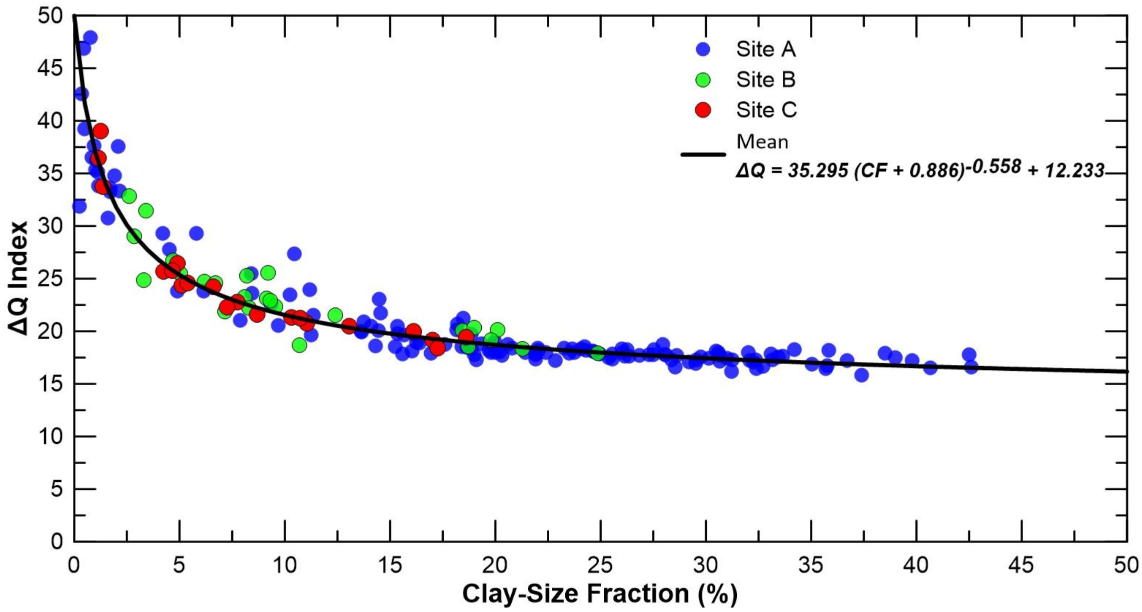

Figure 4. Correlation between ΔQ index and CF from three different sites

ΔQ was also used as an index representing the CPT data, and it was then correlated to the liquid limit of the tailings obtained from grain-size testing in the adjacent soil boring. Figure 5 shows the correlation between ΔQ and the liquid limit developed by the authors. It only shows ΔQ values lower than 39 and liquid limits greater than 19 percent. In general, data points with ΔQ greater than approximately 39 were determined to be NP.

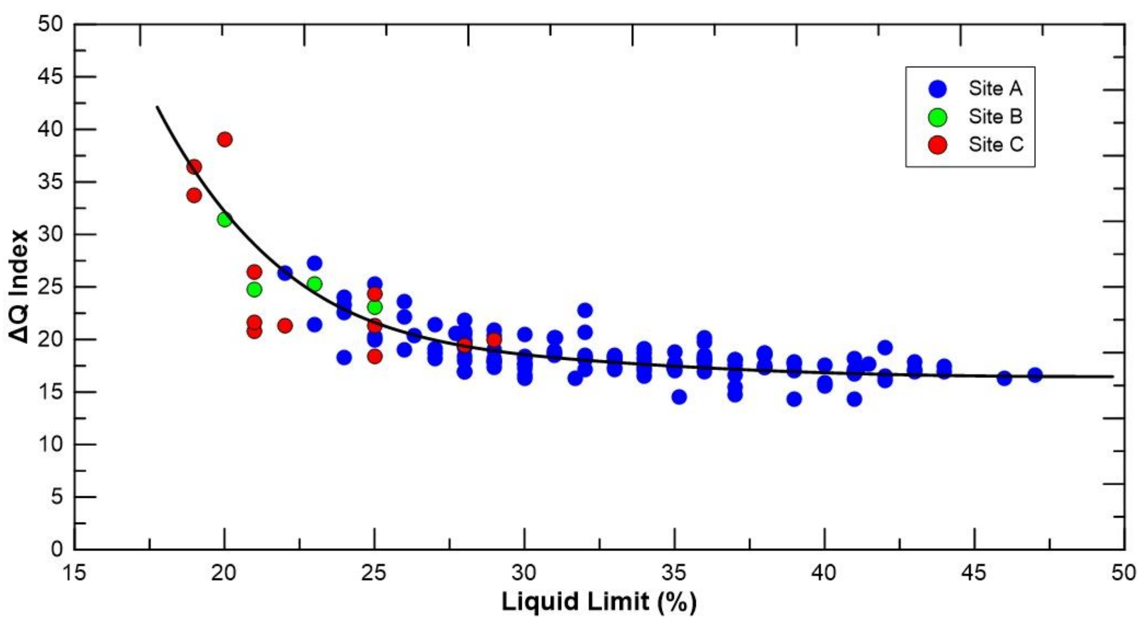

Figure 5. Correlation between ΔQ index and liquid limit from three different sites

6.2 *CPT-Void Ratio*

Figure 6a shows the water content versus depth for the fine tailings collected around the TSF. There is a general trend of water content variability at shallow depths (i.e., at about 6 m) between approximately 25 and 45 percent. As the depth increases, the water content variability still exists but is reduced to the range of 23 and 35 percent. This tendency of the water content to decrease with increasing depth indicates the reduction in void ratio with increasing effective stress. Figure 6b shows this reduction in the void ratio, which has been computed using the specific gravity G_s. It is important to note that the data in Figure 6 do not have an index property for each point.

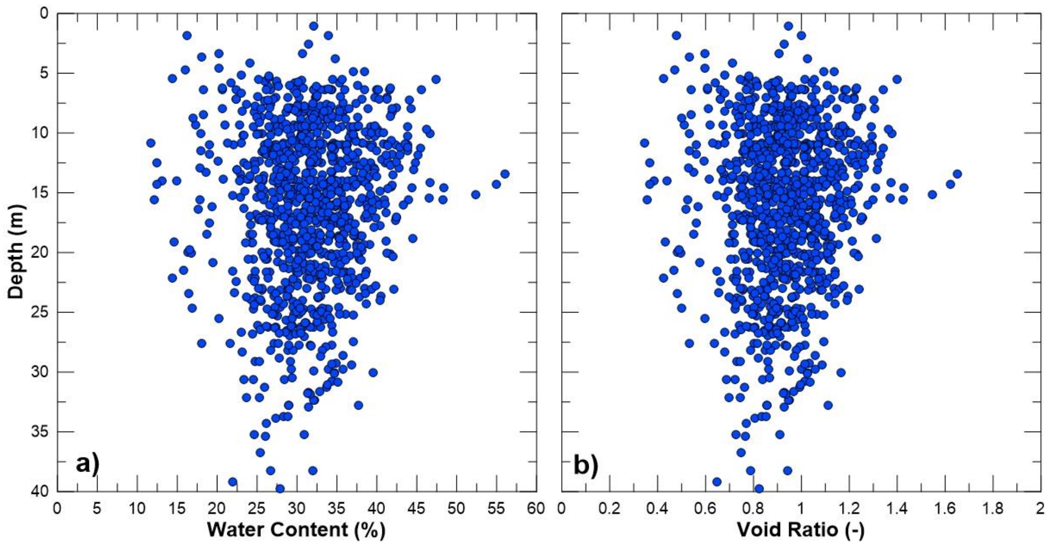

Figure 6. Water content versus depth (a) and void ratio versus depth (b) at the TSF

The data in Figure 6 was used to generate a similar figure with only data points in which CPT soundings were performed, and index properties in an adjacent soil boring were measured. Figure 7a represents the vertical effective stress and water content data, including data points color-coded based on the measured CF. Figure 7a reveals an interesting trend: at a given effective stress, samples with relatively high water content are generally associated with higher CF, whereas points with lower water content generally display lower CF. This is better visualized in Figure 7b, which shows the water content plotted versus the CF and the associated lines of equal effective vertical stress. Figure 7 shows the effects of the effective stress and CF on the measured water content. Figure 7 allows a better understanding of the scatter found in Figure 6.

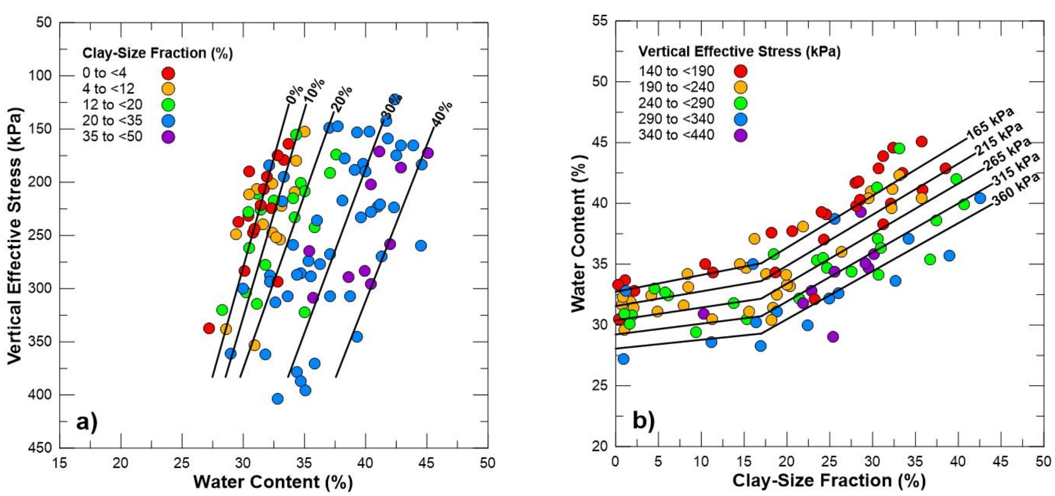

Figure 7. Vertical effective stress versus water content (a) and water content versus CF (b) at the TSF

Figure 8 is an example of the proposed procedure to estimate the in-situ void ratio. Figure 8a shows the ΔQ versus depth for a CPT sounding at the TSF. ΔQ is directly computed using the CPT-measured parameters using equations proposed by Saye et al. (2017). Figure 8b shows the predicted CF from the CPT (using the correlation equation in Fig. 4) and the measured CF (from samples in adjacent soil borings) as colored markers. Figure 8c shows the predicted water contents from the CPT (using the correlation in Fig. 7b) and the measured water content (in samples from adjacent soil borings). Finally, Figure 8d shows the predicted and computed void ratio using the specific gravity and water content (from Fig. 8c and those measured in samples from adjacent soil borings). In general, the specific gravity of the tailings shows some minor increase with increasing CF, and the corresponding value was used in the calculations.

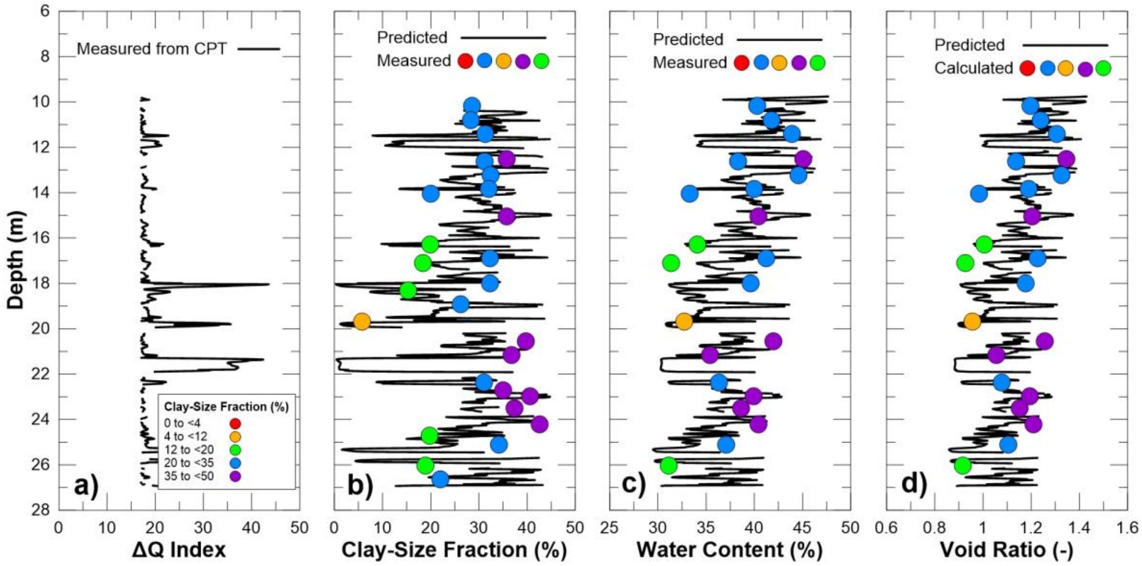

Figure 8. ΔQ index versus depth (a) and comparison of measured and predicted CF (b), water content (c), and void ratio (d) versus depth at TSF

Predicted and measured values in Figure 8 show a good agreement and validate the proposed method. The same procedure was adopted in five other CPT sounding locations with adjacent soil borings and shows similar agreement; however, data are not presented herein due to space limitations.

7 STATE PARAMETER

Figure 9 is an example of the proposed procedure to estimate the state parameter, Ψ. Figures 9a and 9b correspond to the computed ΔQ and void ratio, which are the same as Figures 8a and 8d, respectively. They are shown in Figure 9 to facilitate the explanation. Figures 9c and 9d are developed from the correlation between CF - λ_{10} (Fig. 2b) and CF - Γ (Fig. 2c), respectively. Given the void ratio, λ_{10}, and Γ, the calculation of the state parameter is an algebraic exercise. Figure 9e shows the estimated state parameter Ψ.

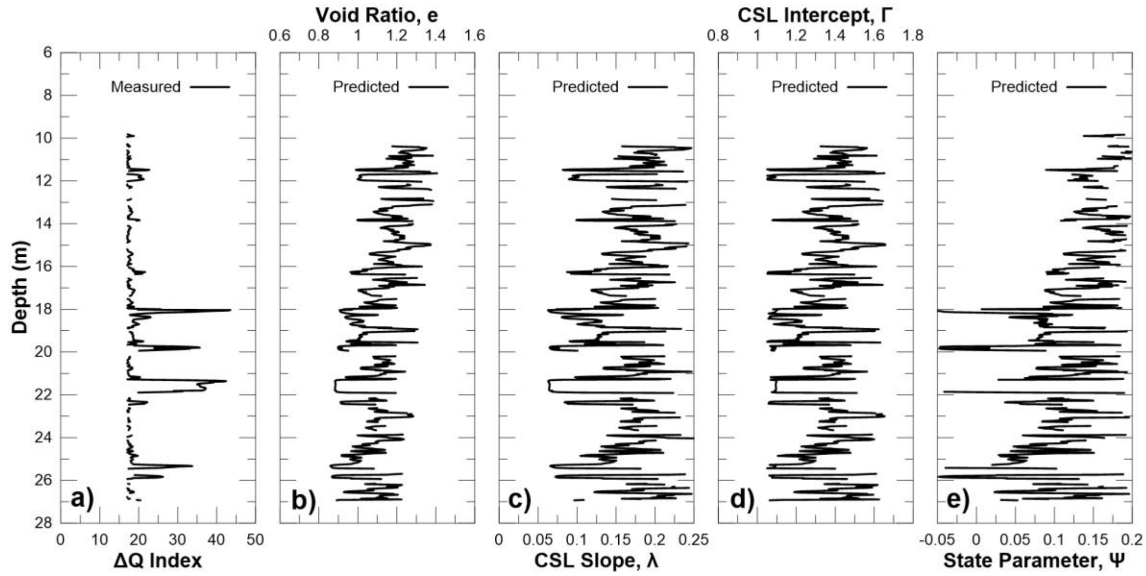

Figure 9. ΔQ index (a), void ratio (b), λ_{10} (c), intercept Γ (d), and state parameter (e) versus depth at TSF

8 UNDRAINED SHEAR STRENGTH

With the estimate of the state parameter, Ψ, it is possible to estimate the undrained shear strength ratio, USSR. Figure 10 is an example of the proposed procedure using data from the same CPT sounding. Figure 10a shows the estimated state parameter from Figure 9e, as previously described. The estimated state parameter profile in Figure 10a is used in conjunction with Figure 3 to predict the yield and liquefied USSR profiles shown in Figure 10b (green and blue lines, respectively). For comparison, estimates of the yield USSR using the empirical cone factor, N_{KT}, method are shown in Figure 10b (black markers). In addition to the USSR profiles, Figure 10b also includes the measured yield USSR using field vane shear testing (FVT).

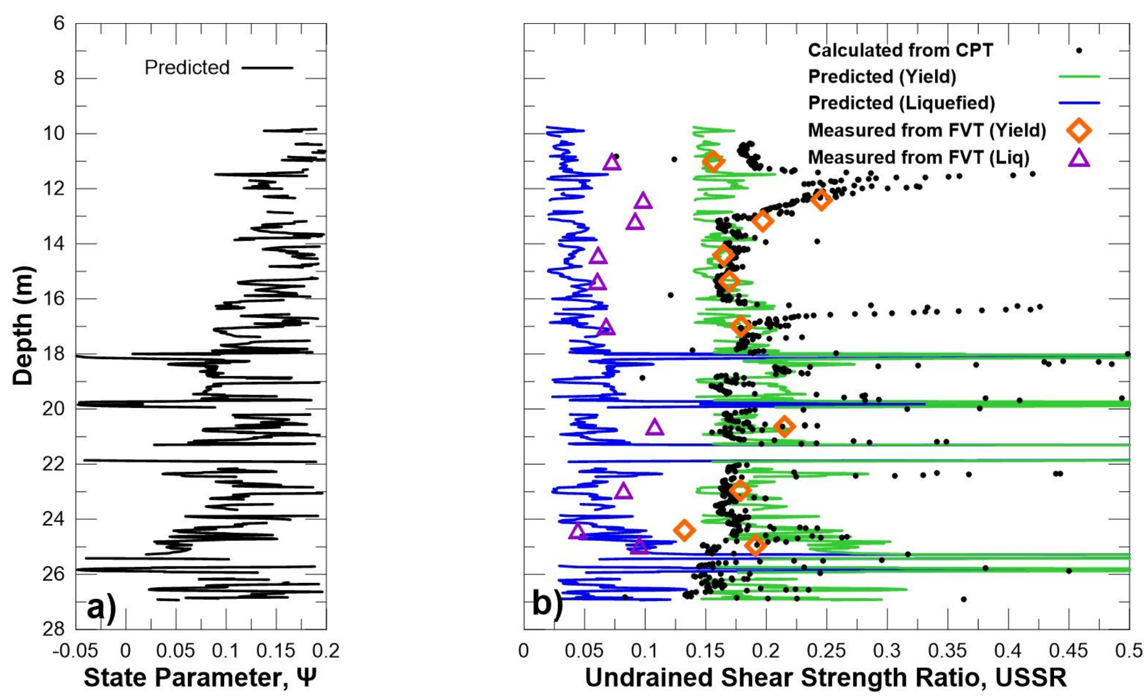

Figure 10 State parameter versus depth (a) and USSR (yield and liquefied) versus depth using the proposed approach, estimated from CPT using the N_{KT} method and directly from FVT (b)

9 SUMMARY AND CONCLUSIONS

This paper presents a brief discussion of the CSSM framework's benefits and challenges as applied to mine tailings. An approach that utilizes laboratory and in-situ testing is proposed to determine the state parameter, Ψ, and USSR for mine tailings. A series of correlations between CPT and index testing derived from soundings and soil borings adjacent to each other are presented and used in the proposed approach. Subsequently, the implementation of the proposed approach is illustrated by using data from one CPT sounding. A detailed description of the approach led to the development of the state parameter, Ψ, and USSR profiles. Finally, a comparison of USSR is shown, with estimations made using the proposed approach, the N_{KT} method, and directly from FVT.

The proposed approach is a good tool for independently verifying other methods for determining USSR in mine tailings, such as Olson and Stark (2002), Robertson (2010), Jefferies and Been (2016), and Contreras and Harvey (2021). However, the applicable correlations are likely to be very material- and site-specific, requiring significant investment in in-situ and laboratory testing and data analysis. These efforts are not undue for TSFs, although implementation may not be practical for fast-paced design efforts. They are, however, highly recommended for verification purposes.

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